



Spin filtering by a periodic spintronic device

Amnon Aharony,^{1,*} Ora Entin-Wohlman,^{1,*} Yasuhiro Tokura,² and Shingo Katsumoto³

¹*Department of Physics and the Ilse Katz Center for Meso- and Nano-Scale Science and Technology, Ben Gurion University, Beer Sheva 84105, Israel*

²*NTT Basic Research Laboratories, NTT Corporation, Atsugi-shi, Kanagawa 243-0198, Japan*

³*Institute of Solid State Physics, University of Tokyo, Kashiwa, Chiba 277-8581, Japan*

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For a linear chain of diamondlike elements, we show that the Rashba spin-orbit interaction (which can be tuned by a perpendicular gate voltage) and the Aharonov-Bohm flux (due to a perpendicular magnetic field) can combine to select only one propagating ballistic mode, for which the electronic spins are fully polarized along a direction that can be controlled by the electric and magnetic fields and by the electron energy. All the other modes are evanescent. For a wide range of parameters, this chain can serve as a spin filter.

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I. INTRODUCTION

In addition to their charge, electrons also carry a spin, which is the quantum relativistic source for the electron's intrinsic magnetic moment. Future device technology and quantum information processing may be based on spintronics,¹ where one manipulates the electron's spin (and not only its charge). One major aim of spintronics is to build mesoscopic spin valves (or spin filters), which generate a tunable spin-polarized current out of unpolarized sources. Much recent research aims to achieve this goal by using narrow-gap semiconductor heterostructures, where the spins are subject to the Rashba² spin-orbit interaction (SOI): in a two-dimensional electron gas confined by an asymmetric potential well, the strength of this SOI can be varied by an electric-field perpendicular to the plane in which the electrons move.³ An early proposal of a spin field-effect transistor⁴ used the Rashba SOI to control the spin precession of electrons moving in quasi-one-dimensional wires. Placed between two ferromagnets, the transport of polarized electrons through such a semiconductor could be regulated by the electric field. However, such devices are difficult to make, due to the metal-semiconductor conductivity mismatch.

Some of the most striking quantum effects arise due to interference, which is best demonstrated in quantum networks containing loops. Indeed, interference due to the Rashba SOI has been measured on a nanolithographically-defined square loop array.⁵ Here we discuss the possibility to construct a spin filter from such loops. Recently, several groups proposed spin filters based on a *single* loop, subject to both an electric and a magnetic [Aharonov-Bohm (AB) (Ref. 6)] perpendicular fields.⁷⁻⁹ However, such devices produce a full polarization of the outgoing electrons only for *special values* of the two fields. In the present paper we consider a chain of such loops, as shown in Fig. 1. The effects of the Rashba SOI on the spectrum of the diamond chain of Fig. 1 were studied by Bercioux *et al.*¹⁰ They found a strong variation of the averaged (over energies) conductance with the strength of the SOI, which they associated with localization of the electron due to interference between different paths in each diamond. Later, this group¹¹ found similar effects due to

both the SOI and an AB flux. However, the possibility to use such networks to achieve spin filtering has not been considered. As we show below, the polarization of the outgoing electrons depends on the energy. Therefore, averaging over energies mixes different polarization directions and eliminates the possibility of obtaining full polarization.

We find that both the ballistic conductance and the spin polarization of the electrons going through the device can be sharply varied by an electric field [determining the SOI (Ref. 3)], a magnetic field (determining the AB phases of the orbital electronic wave functions) and the electrons' energy (set by the chemical potential in the source). Varying these three parameters, we find large parameter ranges where all the energy eigenstates of the device except one become evanescent and decay exponentially, forming the localized states discussed in Refs. 10 and 11. However, the electrons in the remaining single mode propagate with *fully polarized spins*. Thus, electrons which enter with arbitrary spins exit fully polarized. Since this polarization can be tuned by the parameters, our system is an ideal spin filter.

Section II outlines the tight-binding model which we use for solving the Schrödinger equation on the periodic chain of diamonds. Section III presents results for the ballistic conductance and for the polarization of the electrons in the regions where they are fully polarized. Finally, Sec. IV contains a discussion of our results, including a comparison with the case of a single diamond and a discussion of the application of our results to a finite chain.

II. TIGHT-BINDING MODEL

With SOI, we need to solve for the two-component spinor at each point on the network. Bercioux *et al.*^{10,11} treated each bond of the network as a continuous one-dimensional (1D)

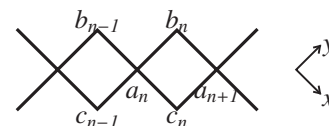


FIG. 1. Chain of diamonds.

wire. Having expressed the solutions along each bond in terms of the spinors of the nodes at its two ends, they used the Neumann boundary conditions at the nodes to derive discrete equations for the spinors at these nodes. As we discuss elsewhere,¹² these boundary conditions are sufficient but not necessary for current conservation at the nodes. A more systematic way to treat such network replaces each continuous bond by a discrete sequence of sites, and then studies the tight-binding model for the wave functions on these sites (and on the original nodes). As the number of these intermediate sites increases, one has more sites per unit cell, and therefore one ends up with more energy bands for the solutions which contain waves moving along the main axis of the network [i.e., along the (1,1,0) direction in Fig. 1]. Qualitatively, we find that all these bands are similar to each other, and also similar to those found for the continuous network used in Refs. 10 and 11. Therefore, we choose to report here only on the simplest case, with no intermediate sites within the bonds. Thus, we treat a simple tight-binding model, with sites $\{u\}$ only on the corners of the diamonds. The latter model could also describe a network of quantum dots or antidots, located at these nodes.¹³ The stationary spinors Ψ_u , with energy ϵ , obey the Schrödinger equations,

$$i\hbar(\partial/\partial t)\Psi_u = \epsilon\Psi_u = -J\sum_v U_{uv}\Psi_v, \quad (1)$$

where the sum is over the nearest-neighbor nodes $\{v\}$, J is the (real) hopping matrix element (in the absence of fields) and

$$U_{uv} \equiv U_{vu}^\dagger = \exp[i(\phi_{uv}^{\text{AB}} + \phi_{uv}^{\text{SO}})] \quad (2)$$

is a unitary 2×2 matrix, representing the phase factors due to the AB flux and to the SOI, ϕ_{uv}^{AB} and ϕ_{uv}^{SO} , respectively. For our structure, all bonds are in the xy plane, and both the uniform magnetic field $\mathbf{H} = H\hat{z}$ and the potential asymmetry which creates the SOI are along the \mathbf{z} axis. As can be seen from Fig. 1, the n th unit cell contains three sites, and Eq. (1) reduces to equations for the related spinors, $\Psi_a(n), \Psi_b(n)$, and $\Psi_c(n)$. Choosing the edges of the diamonds along the x and y axes (see Fig. 1), so that site a_n is located at $\mathbf{r}_n = (n, n, 0)L$ (L is the length of each edge), the unitary hopping matrices within the n th diamond are given by¹⁴

$$\begin{aligned} U_{ab}(n) &= U_{ba}^\dagger(n) \equiv e^{i\phi/2} e^{i\alpha\sigma_x}, \\ U_{ac}(n) &= U_{ca}^\dagger(n) \equiv e^{-i\phi/2} e^{-i\alpha\sigma_y}, \end{aligned} \quad (3)$$

where $\boldsymbol{\sigma}$ is the vector of Pauli matrices, $\alpha = k_{\text{SO}}L$ (k_{SO} measures the strength of the “microscopic” SOI, $(\hbar/m)k_{\text{SO}}\boldsymbol{\sigma} \times \mathbf{p}$) and $\phi = 2\pi HL^2/\Phi_0$ represents the AB phase associated with a single square diamond (here, $\Phi_0 = hc/e$ is the flux unit; h is Planck’s constant, c is the speed of light, and e is the electron charge). Note that the dependence of $U_{ab}(n)$ and of $U_{ac}(n)$ on n results from our choice of gauge for the vector potential. The net flux through each diamond is equal to ϕ , independent of n .

For $\epsilon=0$ one encounters dispersionless modes, for which $\Psi_a(n) \equiv 0$. Since these solutions have zero velocity, and therefore carry no current, we ignore them in the following discussion. We next eliminate the spinors $\Psi_b(n)$ and $\Psi_c(n)$

from the equations, and end up with effective one-dimensional equations,

$$4\Lambda\Psi_a(n) = \mathbf{V}^\dagger\Psi_a(n-1) + \mathbf{V}\Psi_a(n+1), \quad (4)$$

with $4\Lambda = (\epsilon/J)^2 - 4$ and

$$\begin{aligned} \mathbf{V} &= U_{ab}(n)U_{ac}(n+1) + U_{ac}(n)U_{ab}(n+1) \\ &= e^{-i\phi/2} e^{i\alpha\sigma_x} e^{-i\alpha\sigma_y} + e^{i\phi/2} e^{-i\alpha\sigma_y} e^{i\alpha\sigma_x}. \end{aligned} \quad (5)$$

Unlike the individual U_{uv} ’s, the “renormalized” hopping matrix \mathbf{V} is *not* unitary. This lack of unitarity reflects interference between the two paths in a diamond, which may decrease the current along the chain.

In the following we concentrate on propagating waves,

$$\Psi_a(n) = A e^{iq\bar{L}n} \chi_a(q), \quad (6)$$

where $\bar{L} = L\sqrt{2}$ is the lattice constant of the diamond system along its axis (1,1,0), the (real) wave vector q is in the range $-\pi/2 < q\bar{L} < \pi/2$ and χ_a is a normalized spinor (which depends on q). For such solutions, Eq. (4) implies that χ_a must obey the eigenvalue equation $\mathcal{H}\chi_a(q) = \Lambda\chi_a(q)$, with the 2×2 Hermitian matrix

$$4\mathcal{H} = e^{-iq\bar{L}} \mathbf{V}^\dagger + e^{iq\bar{L}} \mathbf{V}. \quad (7)$$

We next write

$$\mathcal{H} = A + \mathbf{B} \cdot \boldsymbol{\sigma}, \quad (8)$$

with

$$A = c^2 \cos(q\bar{L}) \cos(\phi/2),$$

$$\mathbf{B} = -cs \sin(q\bar{L}) \cos(\phi/2) (1, -1, -\cot(q\bar{L}) \tan(\phi/2) s/c), \quad (9)$$

where $c = \cos \alpha$, $s = \sin \alpha$. It follows that the spinor $\chi_a(q)$ must be an eigenvector of the spin component along $\mathbf{n} \equiv \mathbf{B}/|\mathbf{B}|$: $\mathbf{n} \cdot \boldsymbol{\sigma} \chi_a(q) = \mu \chi_a(q)$, $\mu = \pm 1$. Thus, $\Lambda = A + \mu|\mathbf{B}|$. Given Λ , this equation can be written as a quadratic equation in $x = \cos(q\bar{L})$. Denoting the solutions by $x_{1,2}$, we end up with four solutions $q_{1,2}^\pm = \pm \arccos x_{1,2}$. These solutions are propagating (evanescent) if q is real (complex). For each q one then has $\mu = (\Lambda - A)/|\mathbf{B}|$, so that μ is invariant under flipping the sign of q .

Since $\chi_a(q)$ is an eigenvector of $\mathbf{n} \cdot \boldsymbol{\sigma}$, each solution with a given q is associated with a full polarization along the direction \mathbf{n} ,

$$\mathbf{S} \equiv \langle \chi_a(q) | \boldsymbol{\sigma} | \chi_a(q) \rangle = \mu \mathbf{n}. \quad (10)$$

As usual for Rashba SOI, \mathbf{n} is always perpendicular to the direction of motion along the axis of the diamond chain, (1,1,0). In the absence of an AB flux (i.e., $\phi=0$) \mathbf{n} remains in the direction (1, -1, 0). However, the orbital AB flux causes a rotation of the polarization axis toward the \mathbf{z} direction. Below we present results for S_z and for $S_{xy} \equiv (S_x - S_y)/\sqrt{2}$. Since $n_{x,y}$ (n_z) is odd (even) in q , flipping the sign of q flips the sign of S_{xy} but not that of S_z .

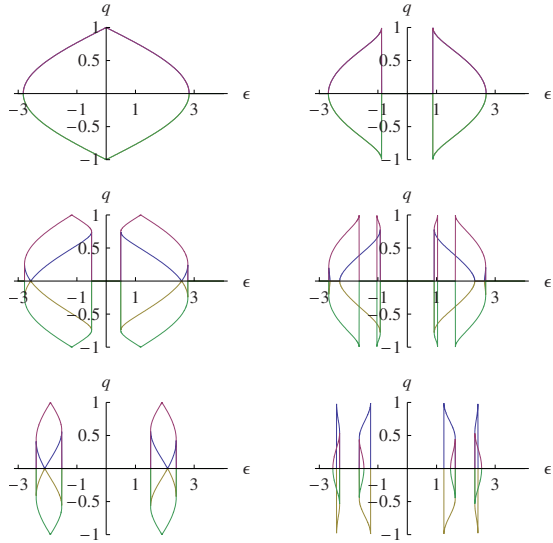


FIG. 2. (Color online) The spectrum (q versus ϵ) of the propagating solutions. Here, $J=1$ and the wave vector q is in units of $\pi\bar{L}$. Left: $\phi=0$. Right: $\phi=0.4\pi$. Top to bottom: $\alpha=0, 0.2\pi, 0.4\pi$. The vertical lines indicate boundaries at which the number of propagating solutions changes.

The probability current from site u to site v is

$$I(u \rightarrow v) = (2J/\hbar)\mathcal{I}\langle\Psi_u|U_{uv}|\Psi_v\rangle. \quad (11)$$

The current from site a_n to site a_{n+1} on the diamond chain, equal to the sum of the currents from a_n to b_n and to c_n , is thus found to be

$$I(n \rightarrow n+1) = -(2J^2/\hbar\epsilon)\mathcal{I}[\langle\Psi_a(n)|\mathbf{V}|\Psi_a(n+1)\rangle]. \quad (12)$$

For a single propagating solution of the form (6), Eqs. (5) and (10) yield

$$\begin{aligned} I(n \rightarrow n+1) = & -(4J^2/\hbar\epsilon)|A|^2 \\ & \times \{\sin(q\bar{L})[\cos(\phi/2)c^2 + \mu n_z \sin(\phi/2)s^2] \\ & + \mu(n_x - n_y)\cos(q\bar{L})\cos(\phi/2)sc\}. \end{aligned} \quad (13)$$

It is easy to see that I flips sign with q . When we have only a pair of propagating modes, we thus concentrate on the one with $I > 0$.

III. RESULTS

Figure 2 shows the spectrum $\epsilon(q)$ of the propagating solutions (real q 's), for several values of ϕ and α . The left column shows results for $\phi=0$, similar to Ref. 10: increasing α splits the energy band vertically, and changes its width. Thus, the SOI can turn propagating waves into evanescent ones, with complex q (our figures show only the solutions with real q). However, whenever the energy ϵ allows for real values of q , there exist four such values, forming pairs which move in opposite directions and have opposite spins along $(1, -1, 0)$.

The situation becomes more interesting when we have both the SOI and the AB flux. Adding only the latter (upper

plot on the right-hand side of Fig. 2) creates a gap (i.e., evanescent states) around $\epsilon=0$. The degeneracy of the propagating solutions is not lifted, since the two spin directions have exactly the same energies. As seen in the right column in Fig. 2, increasing α at fixed $\phi=0.4\pi$ causes the splitting of each subband horizontally.

We next discuss the ballistic conductance of our device, G . For an ideal conductor, this conductance is given by $G = (e^2/h)g$, where g is the number of right-moving (or left-moving) propagating modes at a given energy.¹⁵⁻¹⁷ This formula clearly applies for the infinite periodic chain of diamonds discussed here. Below we argue that the filtering effect which we find also survives for a finite chain, under certain conditions. As Fig. 2 shows, at a given energy ϵ one can encounter zero, two, or four propagating solutions. The number g can be read directly from Fig. 2: on the left-hand side of this figure, the number of real q 's (both left moving and right moving) is always zero or four, and thus $g=0$ or 2. In contrast, the right-hand side of Fig. 2 shows 0, 2, or 4 real q 's, i.e., $g=0, 1$, or 2, depending on the parameters ϵ, ϕ , and α .

We next consider electrons coming with arbitrary spin directions from a reservoir at $-\infty$, with energy ϵ equal to their chemical potential in that reservoir. For each electron, its spinor will become a combination of the eigenmodes of the problem inside the system. In fact, the same will happen to electrons which enter into a *finite* but long chain from the left-hand side: their spinor within the chain will become a similar combination of the four eigensolutions there, multiplied by some transmission coefficients. When all four q 's have nonzero imaginary parts, all of these modes are evanescent, and the wave function will decay to zero, resulting with zero current. In that case there are no propagating modes, and $g=0$. When all four q 's are real, i.e., $g=2$, the incoming wave function is a combination of two right-moving modes, and it has no definite spin. However, for $g=1$ the wave function of the right-moving electron is a linear combination of one propagating and one evanescent modes. The latter will decay, and the spinor will converge to that of the *single propagating solution*, which has a *uniquely polarized spin*, see Eq. (10). Without the AB flux, we always had $g=2$ or $g=0$. For $\phi \neq 0$, we find regions of energy where $g=1$. Figure 3 shows contour plots of g in the ϕ - α plane, for several values of ϵ . As one can see, for energies $\epsilon = -1.2J$ and $\epsilon = -2.4J$ there are large regions where $g=1$. In these regions, the electron will have a well defined polarization, which depends only on ϵ, ϕ , and α .

We next consider specific cuts through these contour plots. Figure 4 shows results as a function of α for fixed energy $\epsilon/J = -2.4$ and AB phase $\phi = 0.29\pi$. The plots show only the ($g=1$ or $g=2$) right-moving modes ($I > 0$). The other propagating modes have opposite signs for q, I , and S_{xy} . The dashed curves represent the second mode, which arises only when $g=2$. For our purposes, we concentrate on the regions where $g=1$, where one has only the dotted lines. The top plots show the solutions for q and the corresponding currents I . The bottom plots show the spin components S_{xy} and S_z . The variation of S_{xy} with α is striking: the spins of the propagating electrons switch the sign of their in-plane component with a small change of α near $\alpha = \pm 0.5\pi$. Note also

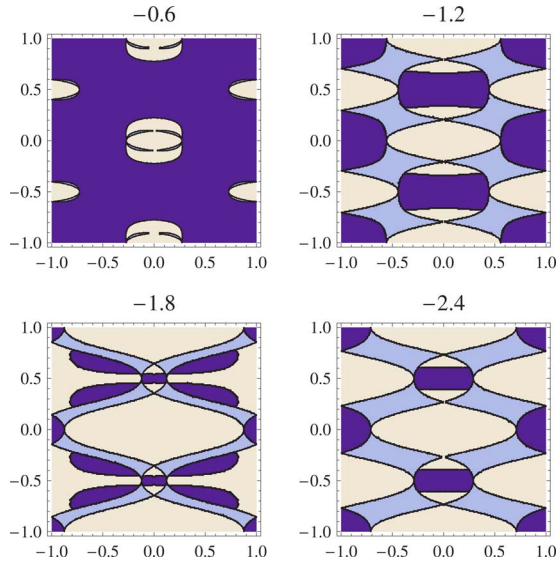


FIG. 3. (Color online) Contour plots of the ballistic conductance (in units of e^2/h) in the ϕ - α plane (the AB phase ϕ and the SO strength α are in units of π). The values $g=0, 1, 2$ are represented by dark, medium, and bright areas. The number above each plot is the energy ϵ (in units of J).

the flipping of S_{xy} as α crosses $\pm\pi$. This flipping persists as ϕ increases, and the range with $g=2$ near these points narrows. Figure 5 shows results as a function of ϕ , for the same energy, but at fixed SO strength $\alpha=0.75\pi$. Clearly, even a relatively small AB flux already yields a single right-moving propagating mode ($g=1$) and therefore fully polarized spins. At small ϕ , the polarization starts close to the $(1, -1, 0)$ direction, but it then rotates toward the z direction as ϕ increases toward $\pm\pi$, and flips sign after crossing these points.

IV. DISCUSSION

Given the above analysis, we may compare our system with that of the single diamond, Ref. 8. As we report elsewhere,¹⁸ the single diamond generates fully polarized

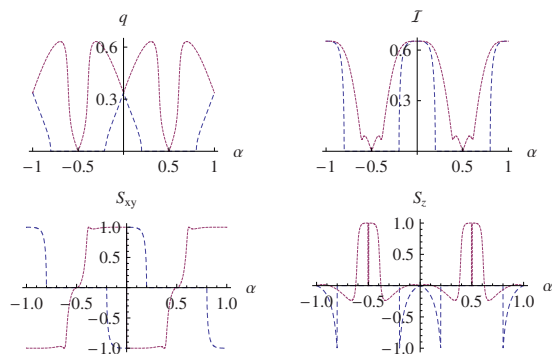


FIG. 4. (Color online) Wave vectors q (in units of $\pi\bar{L}$), currents I , and spin components S_{xy} and S_z , for right-moving modes, as functions of the SO strength α (in units of π), for $\epsilon/J=-2.4$ and $\phi=0.29\pi$. For values of α at which $g=1$, the figures show only one mode (dotted line). When $g=2$, the figures show two modes (dotted and dashed lines).

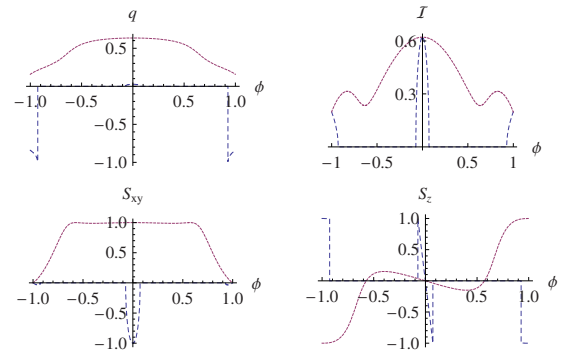


FIG. 5. (Color online) Same as Fig. 4, for $\epsilon/J=-2.4$ and for fixed SO strength $\alpha=0.75\pi$, as functions of ϕ (in units of π).

electrons, along a controllable direction, whenever $\sin^4 \alpha = \cos^2(\phi/2)$ and for any ϵ . Although this condition is less restrictive than that given in Ref. 8, it is still much more restrictive than the conditions we found above. The literature contains many other proposals for spin filters, also based on the Rashba SOI. Usually, these give only a partial polarization. Some of these devices also require a large Zeeman field. In contrast, our filter can work at a relatively low (and fixed) magnetic field (as apparently desired technologically), so that the Zeeman energy is negligible. Note also that both α and ϕ depend on the diamond size L , and therefore one can choose a geometry which corresponds to the available ranges of the magnetic field and the microscopic Rashba parameters.

In real experiments it is not realistic to use an infinite chain of diamonds. We now argue that under appropriate conditions it is sufficient to use a finite chain, as long as it is longer than the decay lengths of the evanescent modes. For the electrons coming in from the left we do not need to worry about the details of the connection between the incoming lead and the chain; even if some of the electrons are reflected back into that lead, those which are transmitted into the chain will split into a sum of the four modes there, and when $g=1$ we still remain with fully polarized electrons (although their overall amplitude may involve a transmission factor with magnitude smaller than 1). The situation on the right-hand end of the chain is more delicate. Here we should avoid reflections, since they may modify the outgoing spinors and change their polarization. A standard way to avoid reflections is to use *adiabatic* contacts. This is usually done for retaining the ballistic conductance of mesoscopic devices.¹⁶ One way to avoid reflections is to have a large leakage to the ground near the exit channel, so that only a small fraction of electrons enter into the exit lead.

For our filter to be useful, one also needs to measure the outgoing spins, or to relate the outgoing spin polarization to some measurement of a voltage or a current. This issue is common to many proposed filters, and it requires separate research. For the present purposes, we mention just a few possibilities. First, one can follow the original proposal of Datta and Das,⁴ and connect the right-hand end of the device adiabatically to a ferromagnetic lead, whose magnetization can be tuned. The outgoing current will decrease with the angle between the electron polarization and this magnetiza-

tion. Second, to avoid connections to ferromagnets, one can also connect our filter adiabatically to another such filter, with different parameters which may block the polarized electrons coming from the first filter.

Another way to test the spin polarization is to couple one of the a nodes (Fig. 1) to a side quantum dot, that is in a Pauli-spin blockade region.¹⁹ After a while, the side dot will capture one of the polarized electrons, and this will block the current (which contains electrons with the same polarization). Changing the parameters will then change the spin direction of the propagating electrons, and allow some current until the next blocking occurs.

In conclusion, we propose a simple spin filter, which yields a full polarization over a broad range of parameters.

For given energy ϵ and magnetic flux ϕ (which need not be very large), the polarization of the outgoing electrons can be tuned by varying the electric field which determines the SOI strength α .

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